BROWN UNIVERSITY Providence, Rhode Island • 02912

DIVISION OF ENGINEERING

N67-31495 (ACCESSION NUMBER) (THRU)
5 (PAGES) 1 (CODE)
CR-85816 (NASA CR OR TMX OR AD NUMBER) 33 (CATEGORY)

FACILITY FORM 602

SC-NGR-40-002-012

FINAL REPORT

April 15, 1965 - April 15, 1967

Variational Methods for Solving Heat Conduction Problems

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Report

Item 1. Use of a transcendental approximation in transient conduction with a non-linear boundary condition

A report, "Use of a transcendental approximation in transient conduction analysis" by P. D. Richardson and W. W. Smith was submitted to NASA in 1965, and an amended version (with previous typographical errors corrected) was transmitted to NASA in 1967. New results were obtained for problems involving transient conduction in a slab with a non-linear boundary condition at the surface, using a variational method and a two-parameter transcendental approximation function. In limiting cases (where the boundary condition was linear) the agreement between the approximate, variational solution and the exact solution was very good: for constant surface heat flux, for example, the solutions differed by about 0.2 per cent.

Item 2. Combined transient conduction and radiation

A report "Transient radiation and conduction in a slotted slab and a hollow cylinder" by P. D. Richardson and Y.-M. Shum, was submitted to NASA Headquarters in June, 1967. In this report, temperature distributions in a parallel-walled channel (a typical slot in a slotted slab) and in a hollow cylinder are considered. Each end of the channel or cylinder faces an environment of uniform but alterable temperature. Black wall surfaces are assumed, and for most solutions presented the radiant heat flux is linearized with respect to temperature. Transient solutions are obtained for cases where the walls are thin, and where they are (thermally) thick. In the latter case, transient conduction normal to the wall surfaces is coupled into the problem. Examples of superposition are given. For the hollow cylinder, prescribed heat generation along the wall is also treated. The concept of the radiation mean effective path is found to be useful. Generalization to non-black surfaces is discussed.

Item 3. Combined transient conduction and radiation: lunar surface cracks

When transient, two-dimensional conduction is considered, it is clearly more difficult to find analytical approximations which can be used with a variational principle as has been done for transient, one-dimensional conduction. The need for fairly simple integration of the approximation adds to the restrictions in the two-dimensional case. In these circumstances, variational-numerical methods are attractive. Recently Gurtin [1] demonstrated variational principles for linear initial value problems, and Wilson & Nickell [2] used this to construct a scheme for approximate, numerical solution by the method of finite elements. In this approximation, the temperature distribution at any specific time (which can be imagined as a surface in (T, x, y) space) is approximated by a surface made up of plane triangular segments which have abutting edges. The temperature distribution is determined by values at the nodal points, between which linear interpolation can be used. Attractions of this method over other numerical schemes include freedom of choice of the elements, the ability to apply boundary conditions at any point within the system, lack of restriction with respect to geometry and material property distribution, and lack of restriction on space and time intervals used in obtaining the solution--either or both can be changed during the solution of a problem.

[1] M. E. Gurtin "Variational principles for linear initial-value problems" Quart. Appl. Math., vol. 12, p. 252 (1964)

[2] E. L. Wilson & R. E. Nickell "Application of the finite element to heat conduction analysis" Proc. Fifth U.S. Natl. Congr. Appl. Mech. p. 841 (1966)

Several problems have been considered, using this method. The first three cases involve problems for which some solution is already available; these cases served to check the use of the method and to verify its advantages. The fourth problem is an original one, with application to orbital surveying of the lunar surface.

The first problem considered was that of a semi-infinite solid, initially at T_i throughout, radiating from its exposed surface to a sourceless, black environment. The results were compared with those of Abarbanel [3], who converted the problem into a singular, non-linear Volterra-type integral equation, and solved the latter by numerical quadrature. The agreement was excellent.

The second case considered was an extension of a previous analysis under this grant: transient radiative cooling of a semi-infinite solid with parallel-walled cavities. This problem had been examined somewhat before by Winter [4] using an ordinary finite difference scheme. Even when a rather large element size was used, the agreement between the two solutions was good. One of the results which Winter demonstrated was that surfaces with cracks in it would give anomalous apparent radiation temperature reading in instruments used to view the surface from different angles, the instruments integrating over an area large compared with a typical crack aperture.

The third case, treated by this method, was the problem of the surface temperature of an idealized moon. It was assumed that the surface is uniformly smooth, black and has certain values for its thermal properties. A region on the lunar equator was considered, and was treated as a semi-infinite slab with the appropriate boundary condition for its exposed face:

$$k\frac{\partial T}{\partial x} = \sigma T^4 - A_o \sin\left(\frac{2\pi}{\tau}\right)t \quad \text{for } 0 \le t \le \tau/2 \qquad \text{(lunar day)}$$

$$k\frac{\partial T}{\partial x} = \sigma T^4 \quad \text{for } \tau/2 \le t \le \tau \qquad \text{(lunar night)}$$

The deep-temperature of the moon must be chosen carefully in order to reduce the number of lunations which have to be computed to achieve a steady periodic solution. The computed results agree very well with those available in the literature.

[3] S. S. Abarbanel "On some problems in radiative heat transfer", MIT OSR TN-59-531 (April 1959)

[4] D. F. Winter "Transient radiative cooling of a semi-infinite solid with parallel-walled cavities", Int. Jl. Heat Mass Trans. vol. 9, p. 527 (1966)

The fourth problem considered is a combination of the two previous problems. The lunar surface is considered again, but instead of being uniformly smooth it has deep cracks in it. These cracks are idealized as semi-infinite slots. Unlike the slots treated previously, however, the opposite sides of the slot are not at the same temperature, and after sunrise the solar radiation is incident upon only face of the inside of the slot, the depth of penetration of solar radiation into the slot increasing as the sun approaches local noontime, and so on. This set of conditions gives rise to a temperature difference between the top edges of the slot of about 10 per cent of the average temperature during the lunar morning and afternoon; near noontime the temperatures become equal momentarily as the solar radiation becomes directly incident upon the second slot wall.

Preliminary calculations have been carried out using a relatively coarse element size, and it is now desired to perform calculations of much-improved accuracy by using a smaller element size.

A practical application of these studies is determination of the effect of lunar surface cracks on the apparent radiation temperature (magnitude and phase) as would be observed from an orbiting lunar surveying vehicle. A detailed account of these calculations and a discussion of the results will be given at a later date.

Item 4. Transient conduction in a passive probe, used to measure adaptive response of human skin heat transfer under sudden change of surface temperature

Real conduction problems are good sources of examples for testing techniques. Measurements made in a passive, solid probe used to examine the response of limited areas of human skin to sudden change of environmental temperature provide data of this sort. Different methods of solution were compared for this, and some results are given in "The Response of Human Skin to Localized Heat Sources" by P. D. Richardson and J. H. Whitelaw, 7th International Conference on Medical and Biological Engineering (1967).

Item 5. Approximation of repeated integrals of the error function

A note describing a method of approximating repeated integrals ofthe error function was included with the semi-annual status report for April - October, 1966. This method is very convenient for calculation by hand or digital computer.

Personnel

The following persons have contributed to the work mentioned in this report:

D. Cygan, Graduate Student -- NASA Trainee
S. H. Kozak, Research Assistant (through August 1965)
P. D. Richardson, Associate Professor (Principal Investigator)
Y. M. Shum, Fellow
W. W. Smith, Graduate Student
J. H. Whitelaw, Lecturer, Imperial College, London (at Brown University, July - September 1965)

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